Competitive Spectrum Trading in Dynamic Spectrum Access Markets: A Price War

Yi Tan
Department of ECE
Stevens Institute of Technology
Hoboken, NJ
Email: ytan@stevens.edu

Shamik Sengupta
Department of Math. & Comp. Sci.
John Jay College of Criminal Justice
CUNY, New York, NY
Email: ssengupta@jjay.cuny.edu

K.P. Subbalakshmi
Department of ECE
Stevens Institute of Technology
Hoboken, NJ
Email: ksubbala@stevens.edu

***Abstract*—The concept of dynamic spectrum access (DSA) enables the licensed spectrum to be traded in an open market where the unlicensed users can freely buy and use the available licensed spectrum bands. However, like in the other traditional commodity markets, spectrum trading is inevitably accompanied by various competitions and challenges. In this paper, we study an important business competition activity – price war in the DSA market. A non-cooperative pricing game is formulated to model the contention among multiple wireless spectrum providers for higher market share and revenues. We calculate the Pareto optimal pricing strategies for all providers and analyze the motivations behind the price war. The potential responses to the price war are in-depth discussed. Numerical results demonstrate the efficiency of the Pareto optimal strategy for the game and the impact of the price war to all participants.**

I. Introduction

The recently proposed dynamic spectrum access (DSA) paradigm allows unlicensed users to opportunistically use the unoccupied spectrum bands on a non-interfering basis [1]. This creates a new open market for trading spectrum by the spectrum owners, providers and end users. In this market, the wireless spectrum providers (WSPs) buy spectrum bands from the spectrum owners (e.g., Federal Communication Commission (FCC) in United States) and sell the spectrum service to the end users. Unlike traditional wireless users who subscribe to a single WSP for a contractual period of time (e.g., 1 or 2 years in the existing markets), the users in DSA networks are spectrum agile, enabled with the newly proposed cognitive radio technology and have the freedom to dynamically choose the WSPs based on their own preferences [1].

Several researchers are investigating the pricing problems in DSA markets (see [2]–[4] and the references therein). However, most of these works concentrate on how to formulate an optimal pricing strategy to maximize the revenue from the perspective of every individual provider without taking the potential *fighting for market share* into consideration. In the real business world, companies often adopt several tactics to capture more market share and beat the competitors, e.g., price war and business information warfare. As an example, the two largest U.S. wireless carriers, AT&T and Verizon, recently have escalated a price war to absorb more customers [5]. This kind of competition is inevitable in an open DSA market and cannot be overlooked. Currently, there is little understanding on how such a dynamic price war will operate so as to make open DSA markets feasible under economic terms.

In this paper, we study the price war in the DSA market among multiple WSPs. Assuming that all providers are independent and profit-seeking, we define a demand function for each individual WSP and formulate a non-cooperative pricing game among them using the profit as payoff. The optimal initial prices for all WSPs are analytically derived and proven to be Pareto optimal. We investigate the price war from two perspectives. In the short-term price war, providers lower their prices to capture more market share and hence increase the profit in the short period. We study the impact of such price war for all participants. In the long-term price war, one or several big providers cut the prices significantly in order to monopolize the entire market in the long run. We propose a cooperative responding strategy for small WSPs to fight against such long-term price war and discuss how to set regulations to avoid collusion among multiple big WSPs. Numerical results demonstrate the efficiency of the Pareto optimal price strategies, the progress of the short-term price war and the effectiveness of the cooperative response to the long-term price war.

The rest of this paper is organized as follows. The system model is discussed in Section II. In Section III, we formulate a non-cooperative pricing game and derive the Pareto optimal pricing strategies for WSPs. In Section IV, we analyze the price wars in the DSA market. Section V presents the numerical results and the conclusions are drawn in the last Section.

II. System Model

We consider one spectrum resource owner and n competitive wireless spectrum providers (WSPs) $S_1, S_2, \cdots, S_n$ in the DSA market. Fig. 1 shows the cyclic relationship among the spectrum owner, providers and end users through the arrows connecting the upper and lower halves. To be specific, the spectrum owner manages a chunk of available spectrum bands, i.e., bands not used by primary incumbents, and the WSPs purchase these spectrum bands and provide service which several end users buy in a limited geographical region. We assume that multiple end users can access the same target WSP without blocking and interfering with each other.

For the provider, S_i, we denote D_i to be the estimated demand of bandwidth requested by the end users and q_i to be the quality of spectrum bands. Due to the opportunistic nature of DSA, each WSP should base their spectrum band marketing preference (spectrum quality) on criteria as diverse as throughput, goodput, probability of primary users' return, vulnerability to denial-of-service attack, etc. The characterization of such spectrum quality index is an important issue in the context of spectrum pricing model. In addition, we denote p_i to be the advertised price per unit of the spectrum bands for provider S_i. The end users in the DSA market could be price-sensitive or quality-sensitive [2]. Let $\theta \in [0, 1]$ and $1-\theta$ represent the price-sensitivity and quality-sensitivity factors for end users respectively. For example, $0.5 < \theta \leq 1$ means that the end users are more concerned about the price than the quality and vice versa.

According to marketing principles [6], the demand function should satisfy the following properties:

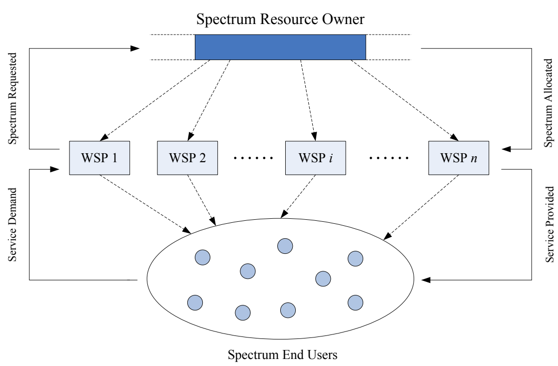


Fig. 1. The DSA market model.

Properties:

1) D_i is a continuous, bounded and differentiable function of p_i and q_i.

2) D_i is decreasing with respect to provider S_i's price, i.e., $\partial D_i/\partial p_i < 0$, and increasing with respect to the other provider's price, i.e., $\partial D_i/\partial p_j > 0$ for all $j \neq i$.

3) D_i is increasing with respect to provider S_i's quality of spectrum bands, i.e., $\partial D_i/\partial q_i > 0$, and decreasing with respect to the other provider's quality of spectrum bands, i.e., $\partial D_i/\partial q_j < 0$ for all $j \neq i$.

4) The total market share is a decreasing function of each individual price $p_i, i = 1, \cdots, n$, i.e., $\partial(\sum_{i=1}^{i=n} D_i)/\partial p_i < 0$, and an increasing function of each individual quality of spectrum bands $q_i, i = 1, 2, \cdots, n$, i.e., $\partial(\sum_{i=1}^{i=n} D_i)/\partial q_i > 0$.

According to the above specifications, we characterize D_i based on the linear price-and-quality dependent demand functions [7], [8] as:

$$\begin{aligned} D_i &= m_i + \theta(-a_i p_i + \sum_{j\neq i} b_{ij} p_j + \pi_i \Delta p_i) \\ &\quad +(1-\theta)(c_i q_i - \sum_{j\neq i} d_{ij} q_j - \gamma_i \Delta q_i), \end{aligned} \tag{1}$$

where the parameters m_i, a_i, b_{ij}, π_i, c_i, d_{ij} and γ_i are positive constants.

In Equation (1), m_i represents the base market share for S_i, which depends on its public reputation, capital, time the WSP existing in the market, etc [6]. a_i and c_i represent the self-influence factors to S_i from itself with respect to the price and quality respectively. b_{ij} and d_{ij} represent the influence to S_i from S_j with respect to the price and quality respectively. Note that b_{ij} and b_{ji} are not equal. We assume that $a_i > b_{ij}$, and $c_i > d_{ij}$ for all $j \neq i$. This assumption indicates that one WSP's demand in terms of the price and quality of spectrum bands is not dominated by any other provider. Δp_i and Δq_i represent the price and quality difference respectively. Let provider S_i change its price from p_i to $p_i^{'}$ or quality from q_i to $q_i^{'}$. Then we define the change in these quantities as $\Delta p_i = p_i - p_i^{'}$ or $\Delta q_i = q_i - q_i^{'}$ respectively.

Let K_i be the cost for provider S_i incurred by purchasing and maintaining the spectrum bands, which depends on the size, l_i, and quality, q_i, of these bands. We also use a linear cost function as:

$$K_i = \alpha q_i l_i + \beta, \tag{2}$$

where α and β are positive constants. It is intuitive that K_i is increasing with respect to l_i and q_i.

Based on Equations (1) and (2), we can derive the profit function, U_i, for provider S_i as follows:

$$\begin{aligned} U_i &= p_i D_i - K_i \\ &= m_i p_i + \theta(-a_i p_i^2 + p_i \sum_{j\neq i} b_{ij} p_j + \pi \Delta p_i) \\ &\quad +(1-\theta) p_i W_i - K_i. \end{aligned} \tag{3}$$

where $W_i = c_i q_i - \sum_{j\neq i} d_{ij} q_j - \gamma_i \Delta q_i$. The secondary order derivation of U_i with respect to p_i yields: $\partial^2 U_i/\partial p_i^2 = -2a_i < 0$, showing U_i is strictly concave with respect to p_i.

III. Non-cooperative Pricing Game

In this section, we look into the pricing strategies for WSPs from the game theoretic perspective. For simplicity, we assume that the spectrum quality for each provider, q_i, is prescribed by the spectrum resource owner and cannot be freely changed.

Considering that every provider is selfish and profit-seeking, we formulate a non-cooperative game among WSPs using the profit as the payoff. Each WSP will choose its own pricing strategy independently so as to maximize their individual payoffs. Note that this game is not a static one-shot game because the WSPs can dynamically change their prices.

In the beginning of the game, the WSPs need to set their initial prices. Since U_i is strictly concave with respect to p_i, the initial price, p_i^*, for player S_i can be obtained by setting $\partial U_i/\partial p_i = 0$ and solving for p_i^*, which gives:

$$p_i^* = \frac{m_i + \theta\sum_{j\neq i} b_{ij} p_j + (1-\theta) Q_i}{2\theta a_i}, \tag{4}$$

where $Q_i = c_i q_i - \sum_{j\neq i} d_{ij} q_i$. From Equation (4), we can see that the optimal initial price for provider S_i, p_i^*, depends on the prices of others. Denoting $P^* = (p_1^*, p_2^*, \cdots, p_n^*)$ to be the optimal initial price vector for all n providers calculated from Equation (4), we have the following theorem:

Theorem: P^* is not the Nash equilibrium but the Pareto optimal strategy set for this pricing game.

Proof:

(i) P^* is not the Nash equilibrium.

In the non-cooperative game, a set of strategies for the players is said to be Nash equilibrium if no player can increase its individual payoff by unilaterally changing its strategy [9].

Consider the scenario where provider S_i changes its price from the optimal initial price, p_i^*, by amount of Δp_i, and all other providers maintain their initial prices. Note that the price difference, Δp_i, is positive when the price is lowered and negative when the price is increased. The new price, $p_i^{'}$, is equal to $p_i^* - \Delta p_i$ and the new estimated demand, $D_i^{'}$, for S_i is given by:

$$D_i^{'} = m_i + \theta(-a_i p_i^{'} + \sum_{j \neq i} b_{ij} p_j + \pi_i (p_i^* - p_i^{'})) + (1-\theta) Q_i. \quad (5)$$

The corresponding payoff for S_i, $U_i^{'}$, is given by:

$$U_i^{'} = p_i^{'} D_i^{'} - K_i, \quad (6)$$

$U_i^{'}$ is also strictly concave with respect to $p_i^{'}$ and so we can calculate the optimal new price, $p_i^{'*}$, to maximize $U_i^{'}$ by solving $\partial U_i^{'} / \partial p_i^{'} = 0$, which is given by:

$$p_i^{'*} = \frac{m_i + \theta(\sum_{j \neq i} b_{ij} p_j + \pi_i p_i^*) + (1-\theta) Q_i}{2\theta(a_i + \pi_i)}, \quad (7)$$

Based on Equations (4) and (7), we have:

$$\begin{aligned} p_i^* - p_i^{'*} &= \frac{\pi_i (m_i + \theta(-a_i p_i^* + \sum_{j \neq i} b_{ij} p_j) + (1-\theta) Q_i)}{2\theta a_i (a_i + \pi_i)} \\ &= \frac{\pi_i D_i(p_i^*)}{2\theta a_i (a_i + \pi_i)}, \end{aligned} \quad (8)$$

where $D_i(p_i^*)$ is the estimated demand for S_i using the initial price p_i^*, which is always positive. Also, the denominator is made up of positive terms only. Hence, $p_i^* - p_i^{'*} > 0$. Since $U_i^{'}$ is strictly concave and has an unique maximum, we see that $U_i^{'*}$ is the maximum payoff for S_i and $U_i^{'*} > U_i^*$.

Therefore, S_i can obtain greater payoff by lowering its price with $\Delta p_i = p_i^* - p_i^{'*}$ unilaterally and thus, P^* is not the Nash equilibrium for this game.

(ii) P^* is the Pareto optimal strategy set.

By definition, an outcome of a game is Pareto optimal if any strategy change that makes one player better off must necessarily make someone else worse off [9].

From *(i)*, we know that given the optimal strategy set P^*, provider S_i can increase its payoff by cutting the price from p_i^* to $p_i^{'*}$ if others maintain their prices the same. However, based on Property 2, we know that the estimated demand for the other provider, S_k, will decline if S_i reduces its price. Since S_k does not change the price, its payoff will also decrease because of the reduction in the demand. Hence, the improvement of the payoff for S_i is associated with the decrease in other providers' payoffs. In other words, player S_i cannot increase its payoff without hurting other players. Thus, P^* is the Pareto optimal strategy set for the game. ■

IV. Price War

Based on the analysis in the previous section, in the beginning of the game, all players will set their initial prices following the Pareto optimal strategy set, P^*. However, since the Pareto optimal point is not the equilibrium, the players who are greedy and selfish also have incentives to markdown their prices to capture more market share and increase the payoffs even though this will harm others' interests. The price war occurs when one or multiple WSPs lower their prices, resulting in similar actions by others. In this section, we will study the price war in the DSA market.

We consider two types of price wars:

- *Short-term price war*: Some providers lower their prices in moderation in order to maximize the profit in the short term and others also lower the prices to match.
- *Long-term price war*: Some big WSPs with high market share drastically cut prices in order to absorb much more market share with the long-term goal of monopolizing the entire market and others try to fight against it.

For both scenarios, we will investigate the motivations and impacts behind the price wars, as well as the responses of participants.

A. Short-term Price War

In the short-term price war, we consider one typical WSP, S_i, to be the first to cut price. From Section III, we know that S_i can maximize its payoff in the short term by cutting its price to $p_i^{'*}$ following Equation (7). Thus, being a rational player, provider S_i will lower the price from the initial Pareto optimal price p_i^* to $p_i^{'*}$ because further price cut will decrease the profit. Provider S_i will gain a larger market share and profit if other providers ignore its deviation and take no actions. This happens in the real market scenario where other WSPs either think it is just a simple promotional activity or do not want to be involved in the price war. However, since the market share and profits of other providers will shrink because of the price cut of S_i, it is also reasonable for them to respond accordingly, i.e., lowering the price to match. Hence, if provider S_k decides to respond, it will also follow the best price cutting strategy based on Equation (7) as:

$$p_k^{'*} = \frac{m_k + \theta(b_{ki} p_i^{'*} + \sum_{j \neq i, j \neq k} b_{kj} p_j + \pi_k p_k^*) + (1-\theta) Q_k}{2\theta(a_k + \pi_k)}, \quad (9)$$

where p_k^* is the Pareto optimal initial price for provider S_k.

As more and more players make responses, a price war occurs among the providers until another Pareto optimal point, $P^{'*} = (p_1^{'*}, p_2^{'*}, \cdot, p_n^{'*})$, is reached. As a result, since all players lower their prices, based on Equation (3), we can see that both initiators and followers of the price cut cannot benefit from the price war but rather lose profit. Afterwards, if someone continues to cut the price, another round of price war will start again and the profits for all WSPs will further decrease.

From the *perspective of end users*, this short-term price war is good because they can take advantage of lower prices to use licensed spectrum bands. However, from the *perspective* *of WSPs*, they will lose profits in the price war and hence the WSPs will eventually end this price war when they reach the point of no returns. Therefore, the short-term price war is self-limiting at some Pareto optimal point. In Section V, we will conduct the numerical analysis to investigate the process and outcome of the short-term price war.

B. Long-term Price War

Sometimes in the DSA market, some big WSPs with high market bases are willing to take the risk of losing profit by drastically cutting prices in order to absorb as much market share as possible. Their long-term goal is to monopolize the entire market in the future, and because of their initial size, they are fairly immune to temporary loss of profits. Eventually, this will result in some small WSPs with low market base to be forced to exit this market because they lose many end users and cannot sustain huge loss in profit. On the other hand, to recover the loss in the price cut, the large WSPs monopolizing the entire market will eventually raise their prices again, even greater than the previous prices.

There is no doubt that this kind of price war will undermine the normality of the DSA market because it leads to the monopolization and without appropriate competitions, the market cannot guarantee the high service level and reasonable price to end users. Hence, it is necessary to take measures to prevent such price war. Here, we discuss two different scenarios.

1) The biggest WSP launches the price war:

In this case, the WSP with the largest market base tries to monopolize the DSA market by cutting the price significantly. Without loss of generality, let provider S_i be the biggest WSP who cuts the price by t (Typically, t is larger than $p_i^* - p_i^{'*}$). If the other WSPs do not respond to this price cut, they will lose many end users, which may threaten their survival. However, price cutting from a small WSP individually may not work because its market base and influence are much weaker than the biggest WSP. Hence, the most effective way out for them is to cooperate to compete with S_i.

If the other providers take coordinated action to prevent S_i capturing the end users from them, provider S_i will not gain any benefits from this price war and may call off the price cut. At the same time, they will also try to minimize the loss in the profit during the competition. Thus, the effective countermeasure is that all other providers lower their prices simultaneously such that the demand for S_i after it reduces the price, $D_i(p_i^*-t)$, remains the same as before, $D_i(p_i^*)$, and their total loss in profits can be minimized. More specifically, denote Δp_j to be the price cut for provider $S_j, j \neq i$, from Equation (5), the demand for provider S_i after the price cut, $D_i(p_i^* - t)$, is expressed as:

$$D_i(p_i^* - t) = m_i + \theta(-a_i(p_i^* - t) + \sum_{j \neq i} b_{ij}(p_j - \Delta p_j) + \pi_i t) + (1-\theta)Q_i. \quad (10)$$

Imposing $D_i(p_i^* - t) = D_i(p_i^*)$, we have:

$$D_i(p_i^* - t) - D_i(p_i^*) = \sum_{j \neq i} b_{ij}\Delta p_j - (a_i + \pi_i)t = 0 \quad (11)$$

Moreover, denote $D_j(p_j^* - \Delta p_j)$ to be the demand for provider S_j, the corresponding profit, $U_j(p_j^* - \Delta p_j)$, is given by:

$$U_j(p_j^* - \Delta p_j) = (p_j^* - \Delta p_j)D_j(p_j^* - \Delta p_j) - K_j. \quad (12)$$

Therefore, the cooperative response can be described as:

$$\max_{\Delta p_j \in (0, p_j^*)} \left[\sum_{j \neq i} U_j(p_j^* - \Delta p_j) \right],$$
$$\text{subject to} \sum_{j \neq i} b_{ij}\Delta p_j - (a_i + \pi_i)t = 0. \quad (13)$$

This nonlinear programming optimization problem can be solved numerically. As a consequence, if all other WSPs cooperatively adopt the strategy described in Equation (13), the biggest provider, S_i, cannot increase its market share by the price cut but will additionally suffer the loss in profit.

2) Multiple big WSPs collude to launch the price war:

In the open DSA market, it is technically possible for WSPs to communicate with each other. Hence, the collusion among big WSPs cannot be overlooked.

Let us consider the scenario where multiple big WSPs collude to launch a price war to capture most users and force some small WSPs out of this DSA market. Since the combination of multiple WSPs always dominates the market, it is not difficult for them to monopolize the entire market. In this scenario, the cooperation of the small WSPs to combat this collusion may no longer work because they are not capable of competing with multiple big WSPs. Under this situation, it is necessary for the spectrum resource owner to set a regulation to avoid such price war. For example, the spectrum resource owner can set a price threshold, p_0, and if the prices of big WSPs are lower than this threshold, the spectrum resource owner will stop selling the spectrum bands to them. The value of the price threshold, p_0, should be set appropriately to make both big WSPs and small WSPs coexist in the DSA market. The determination of p_0 is not within the scope of this paper.

V. Numerical Results

In this section, we present results of the price wars through numerical analysis. We consider three WSPs (S_1, S_2, S_3) and assume $S_1 > S_2 > S_3$ in terms of the base market and influence. Let $\theta = 0.8$, meaning that the end users are more concerned about the price than quality. The parameter values used in the analysis are given in Table I. Values of α and β in Equation (2) are set to 8 and 500 respectively.

TABLE I
The Parameter Values in the Numerical Analysis

WSP	m_i	a_i	b_{ij}	π_i	c_i	d_{ij}	q_i	l_i
S_1	200	12	(7.5, 7)	5	100	(25, 20)	1.0	500
S_2	140	11	(8, 7.5)	5	90	(30, 25)	1.1	440
S_3	80	10	(8.5, 8)	5	80	(35, 30)	1.2	380

A. Pareto Optimal Strategy

First, we calculate the initial Pareto optimal prices for the providers based on Equation (4) as: $p_1^* = 28.1735$, $p_2^* = 28.4245$ and $p_3^* = 28.6935$. Fig. 2 shows that while provider S_1 lowers its price to increase the profit, the profits of other two WSPs will monotonically decrease if they maintain their initial prices. This illustrates the efficiency of Pareto optimal strategies. Moreover, each curve in Fig. 3 represents the profit of one particular WSP who lowers its price while others maintain their initial prices. As shown, every provider has a maximum profit that can be achieved by lowering price unilaterally, which could be the incentive to launch price wars.

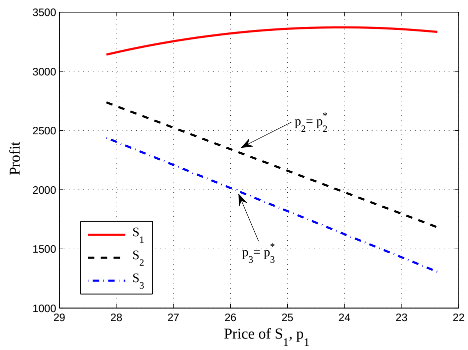


Fig. 2. The change of profits for each WSP when the price of S_1 is lowered from its Pareto optimal initial price.

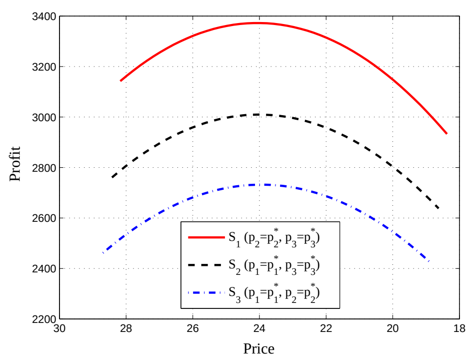
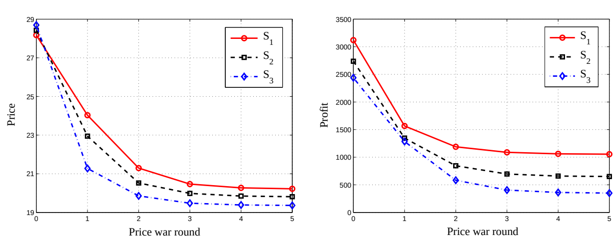


Fig. 3. The maximum profit with respect to the price for each WSP.

B. Short-term Price War

Without loss of generality, let the biggest WSP, S_1, be the first to cut the price following Equation (7) and then S_2 and S_3 respond as Equation (9). Fig. 4 shows the change of price and profit for each provider. As illustrated, the price cut results in loss of profits for all WSPs from one round to the next. Hence, it is reasonable to stop cutting price further at some Pareto optimal point in order to avoid the further loss in profit.

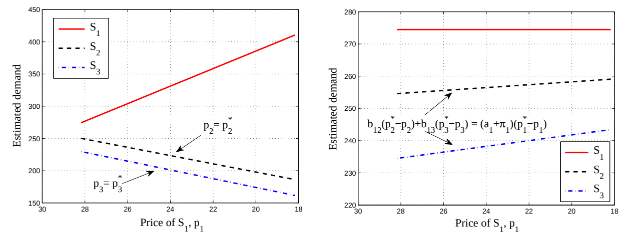


(a) (b)

Fig. 4. The change of prices and profits for the WSPs in the short-term price war. (a) Price; (b) Profit.

C. Long-term Price War

In the long-term price war, the biggest provider, S_1, will significantly reduce its price to capture as many end users as possible. Fig. 5 shows the estimated demands for three WSPs when S_2 and S_3 (a) do not respond and (b) cooperatively respond to the long-term price war. It is observed that if S_2 and S_3 maintain initial prices, they will lose many end users, while simultaneously S_1 will acquire much more market share as it wishes. However, if S_2 and S_3 cooperate to respond based on Equation (13), S_1 cannot increase the market share, while at the same time, the demands of S_2 and S_3 will steadily increase due to Property 4. Thus, S_1 will eventually call off the price cut because this collaborative response makes it gain nothing but enhances its competitors' market share.



(a) (b)

Fig. 5. The estimated demands for the WSPs in the long-term price war. (a) No response; (b) Cooperative Response.

VI. Conclusion

In this paper, we investigated the impact of price wars in DSA markets. Assuming that multiple independent WSPs compete in the market, we modeled a non-cooperative pricing game using the profit as the payoff. The Pareto optimal price strategies for all WSPs were analytically derived and proven. Then, we studied two different types of price wars and revealed the motivations behind them. We also proposed a cooperative countermeasure for the small WSPs to respond to the long-term price war launched by the biggest WSP with the monopolization attempt. Through the numerical analysis, we corroborated the efficiency of the Pareto optimal strategies, demonstrated how the price war starts and proceeds, and showed the potential responses to the price wars.

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