MAT 241 Section 07 Fall 2009
Problem Set 5
Assigned 9/21/09
Due 9/29/09 (Tuesday!)

§2.4: 1, 5, 9, 18

1. Use the graph of $f(x) = 1/x$ (see 2.4.1.png) to find a number δ such that if $|x - 2| < \delta$, then $|1/x - 0.5| < 0.2$.

5. Use a graph of $y = \tan(x)$ to find a number δ such that if $|x - \pi/4| < \delta$, then $|\tan(x) - 1| < 0.2$.

9. Given that $\lim_{x\to\pi/2} \tan^2(x) = \infty$, illustrate Definition 6 by finding values of δ that correspond to (a) $M = 1000$ and (b) $M = 10000$.
[[Recall that Definition 6, the precise definition of an infinite limit, states:
Let f be a function defined on some open interval that contains the number a except possibly a itself. Then:

$$\lim_{x\to a} f(x) = \infty$$

means that for every positive number M, there is a positive number δ such that if $0 < |x - a| < \delta$, then $f(x) > M$.]]

18. Prove the statement using the ϵ, δ definition of limit:

$$\lim_{x\to 4}(7 - 3x) = -5$$

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§2.5: 3, 4, 15, 21, 24, 31, 32, 47; Bonus: 41

3. (a) From the graph of f (see 2.5.3.png) state the numbers at which f is discontinuous and explain why.
(b) For each of the numbers stated in part (a), determine whether f is continuous from the right, from the left, or neither.

4. From the graph of g (see 2.5.4.pmg) state the intervals on which g is continuous.

15. Explain why the function is discontinuous at the given number a. Sketch a graph of the function:

$$f(x) = \ln|x - 2| \quad a = 2$$

21. Explain, using Theorems 4, 5, 7, and 9, why the function is continuous at every number in its domain. State the domain.

$$F(x) = \frac{x}{x^2 + 5x + 6}$$

[[Recall that Theorem 4 states when sums, differences, products, and quotients of continuous functions are continuous; Theorem 5 states that polynomials are continuous over $(-\infty, \infty)$ while rational functions are continuous over their domains; Theorem 7 states that all of the usual functions we deal with are continuous; and Theorem 9 states that the composition of two continuous functions is continuous over its domain.]]

24. Explain, as in 21., why the function is continuous at every number in its domain. State the domain.

$$h(x) = \frac{\sin(x)}{x+1}$$

31. Use continuity to evaluate:

$$\lim_{x \to 4} \frac{5 + \sqrt{x}}{\sqrt{5} + 4}$$

32. Use continuity to evaluate:

$$\lim_{x \to \pi} \sin(x + \sin(x))$$

47. Use the Intermediate Value Theorem to show that there is a root of the given equation in the specified interval.

$$x^4 + x - 3 = 0, \quad (1, 2)$$

[[Recall that the Intermediate Value Theorem states:
Suppose that f is continuous on the closed interval $[a, b]$ and let N be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there exists a number c in (a, b) such that $f(c) = N$.]]

Bonus: 41. For what value of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \geq 2 \end{cases}$$