

MAT 241 Section 07 Fall 2009

Exam I Review Sheet

Exam Date 10/14/09

Review Session TBA

The first exam covers all material in Chapter 2. In addition, it may be useful to review the material from Chapter 1.

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### A. Secant Lines

For problems 1-3, a function and a point on its graph are given. Sketch the graph, and draw a secant line passing through the given point and another point on the graph of your choosing. What is the slope of the secant line?

1.  $f(x) = x^2 + x$     $(1, 2)$
2.  $g(x) = \sin(x)$     $(\pi, 0)$
3.  $h(t) = \ln(t) - 2$     $(1, -2)$

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### B. Limits

For problems 4-6, sketch a function that satisfies each of the given equalities.

4.  $\lim_{x \rightarrow 0} f(x) = 1$     $f(0)$  is undefined
5.  $\lim_{x \rightarrow 1^+} g(x) = -1$     $\lim_{x \rightarrow 1^-} g(x) = 1$     $g(1) = 1$
6.  $\lim_{x \rightarrow 2^+} h(x) = \infty$     $\lim_{x \rightarrow 2^-} h(x) = -\infty$     $h(2) = 0$

In problems 7-9, use limit laws to evaluate each limit.

7.  $\lim_{x \rightarrow -1} 2x(x + 1)$
8.  $\lim_{x \rightarrow 2} \frac{x^2 + 1}{x^2 - 1}$
9.  $\lim_{t \rightarrow 3} \sqrt{t^3 - 9}$

In problems 10-12, find the one-sided limits of the function at the given point. Use this to sketch the graph of the function over the given interval.

10.  $f(t) = -\frac{1}{t^2}$     $t = 0$     $[-1, 1]$
11.  $g(x) = \frac{1}{x^2 - 1}$     $x = 1$     $[0, 2]$
12.  $h(x) = \ln(x + 2)$     $x = -2$     $[-4, 0]$

In problems 13-15, a function is given. Evaluate the limit of each function at  $\infty$  and  $-\infty$ . What geometric features of the graph of the function are described by these limits?

13.  $f(x) = \frac{x + 1}{x - 1}$
14.  $g(t) = 2 \arctan(t)$
15.  $h(x) = e^{-x}$

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### C. Continuity

16. What sorts of discontinuities are exhibited by the functions in problems 4-6?

In problems 17-19, determine where the given function is continuous..

17.  $f(t) = \frac{\sqrt{t-1}}{t-2}$

18.  $g(x) = \sqrt{x^2+1}$

19.  $h(t) = \ln(\sin(t))$

In problems 20-22, use continuity to evaluate the given limit.

20.  $\lim_{x \rightarrow 1} (x^2 + x) \ln(x)$

21.  $\lim_{x \rightarrow -\pi} \frac{\sin(x) + \tan(x)}{\cos(x)}$

22.  $\lim_{x \rightarrow 1} \ln(\sin(2 \arctan(x)))$

In problems 23 and 24, use the Intermediate Value Theorem to show that, given  $f(x)$ ,  $[a, b]$ , and  $N$ , there is  $a < c < b$  with  $f(c) = N$ .

23.  $f(x) = \sin(x)$   $[a, b] = [0, \pi/2]$   $N = 1/7$

24.  $f(x) = x^3 - 2$   $[a, b] = [1, 2]$   $N = 0$

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#### D. Derivatives

In problems 25-27, draw a graph of the function on the given interval, and then sketch its derivative.

25.  $f(x) = \sin(x)$   $[-\pi, \pi]$

26.  $g(t) = \tan(t)$   $(-\pi/2, \pi/2)$

27.  $h(x) = \frac{1}{1+x^2}$   $[-1, 1]$

In problems 28-30, evaluate the derivative of the given function at the given point.

28.  $f(x) = x^3$   $x = 2$

29.  $g(x) = \sqrt{x+1}$   $x = 0$

30.  $h(x) = \frac{1}{x^2}$   $x = -1$

In problems 31-33, evaluate the derivative of the given function.

31.  $f(x) = \frac{x^2}{2}$

32.  $g(x) = \sqrt{2x}$

33.  $h(t) = \frac{1}{t+1}$